ADDRESSING DIFFICULTY IN CALCULUS LIMITS USING GEOGEBRA

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ABSTRACT: *This paper aims to address the difficulties of high school students in bridging their computational understanding with their visualization skills in understanding the notion of the limits in their calculus class. This research used a pre-experimental one-group pretest-posttest design research on 62 grade 10 students enrolled in the Science, Technology, and Engineering Program (STEP) in one of the public high schools in Zamboanga del Sur, Philippines. A series of remedial sessions were given to help them understand the function values, one-sided limits, limits of a function, and its discontinuities with the aid of the GeoGebra. Results revealed that there is a significant improvement in their mathematics performance before and after the remediation, $t(61) = -19.99$, $p<.05$. The real-time feedback provided by the drawing interface of GeoGebra has provided the students the insights into how objects behave in geometrical space, bridging their understanding in locating the values shown by the computational algebraic processes. Additionally, the significant difference in the achievement reveals a large effect size (Cohen's $\delta=2.54$) which could be accounted for by the remediation using interactive activities in the GeoGebra.*

Keywords: GeoGebra limits in calculus, mathematics performance, pre-experimental design

1. INTRODUCTION

One of the pressing challenges facing DepEd's Special Science Curriculum is improving students' outcomes in advanced subjects, especially in applied sciences and advanced Mathematics courses. These subjects are offered to produce highly competent individuals ready for any kind of challenge at the college level, especially in science, mathematics, and all research-oriented courses. Many economic theorists believe that certain types of education equip a man to perform a specific job or function or enable him to perform that function effectively because it improves one's ability to receive, decode, and understand information, as well as the processing and interpretation of information, which is necessary to perform or learn to perform a variety of jobs [1].

The Zamboanga del Sur National High School (ZSNHS), Science & Technology Engineering Program (STEP) curriculum in Grade 10 has been offering Calculus as an elective mathematics subject aside from the basic math course offered by the K-12 program. Scholastic reports have shown that a big number of students have been facing difficulties in dealing with concepts on limits in Calculus. Historical data of students in this curriculum have obtained only 58.73% correct frequency average on the 15 items on the graphical evaluation of limits. Their graphical task competence on limits was low even though those students had correctly responded to the computational evaluative skills and definitions of limits.

From the previous year, some difficulties and hindrances of the students were observed in dealing with limits: poor understanding of the coordinate system; difficulties with the algebra of signed numbers; low understanding of the translation of function graphs; some students are memorizing the concept without understanding; and anxiety towards the subject. There seem to have inherently difficult topics on these subjects which confuses students no matter how these are taught by the teachers [2]. If these students are not provided with proper support to overcome these challenges, they may have difficulties in understanding prerequisite topics, thus facing difficulties in their college courses, and will be gradually dropped out of school before completing their formal education.

Several studies have found that students who use computer-based learning enjoyed Mathematics more. They appreciate the computer practice's flexible characteristics, spend lengthy hours at a computer to finish a job, and love testing out new ideas on the computer. They are no longer passive receivers of information; instead, they participate actively in knowledge building. Thus, rather than simply delivering information, the computer's computational capability is leveraged to support those operations. ICT-based learning activities improved the classroom environment, students are more ready, collaborative, engaged, and autonomous [3].

According to studies, models and simulations in the classroom using GeoGebra helped pupils enhance their knowledge of mathematics lectures. The use of this software was able to facilitate the task of the teacher to provide a constructivist way of developing knowledge beneficial to them [4]. In the interactive constructive environment of GeoGebra, the students have access to generalizing conjectures by clicking and dragging geometrical objects, which dynamically simulate and update information on the screen as they drag the mouse. The models and simulations that may be developed from these settings to display visuals for concepts of limits in Calculus may bridge the gap between the abstract notion of limits and the evaluative measures used in the actual algebraic process. In this way, students would be able to conceptualize the abstract ideas of Mathematics lessons using this environment. This notion provides a basis for how remedial sessions involving modeling and simulations in GeoGebra become a good learning assistant tool in a remedial session and how active learning provides students opportunities to enhance their understanding of the concepts of limits.

The result of this study would provide information and insights about the integration of modeling and simulations of GeoGebra in the classroom as a significant development in the mathematics education process. The study would have meaningful value to the students, parents, school administrators, teachers, and DepEd officials. The students who are ultimate beneficiaries of this study could enhance their content knowledge in Mathematics while simultaneously developing their information technology skills through meaningful and purposive activities.

The teachers would be encouraged to maximize the use of computer-based instructions to improve the mathematics performance of the students. Through the process of integration of the GeoGebra, the teachers could get fast feedback on what the students learn and what lessons need an immediate enrichment discussion. This integration could develop the skills of the teachers to utilize effectively the available information technology equipment in school. Teachers' ability on resourcefulness could be enhanced also for the modeling and simulations to have infinite downloadable instructions from the web domain.

2. MATERIAL AND METHODS

The study was delimited in investigating the effects of remedial classes using modeling and simulations in GeoGebra on the mathematics achievement of the students. This was conducted in Zamboanga del Sur National High School (ZSNHS), Sta. Maria District, Pagadian City. 62 grade 10 students (30 males and 32 females) from the Science & Technology Engineering Program (STEP) curriculum participated in the remedial sessions. The attendances of the respondents were regularly checked during the two (2) weeks of remediation. Necessary permits and communications for the implementation of this study complied. Letters of permission and consent were secured from the principal’s office, the head teacher of the math department, the grade 10 curriculum head, class advisers, and the students themselves.

The concepts of limits and continuity were the focus of the study, particularly the introductory lessons of Calculus in the elective math of the curriculum. The topics are the definition of limits, rules on limits, one-sided limits, infinite limits, and continuity of limits. The remedial classes on these concepts were designed with the integration of the generative learning environment of Dynamic Geometry Software (DGS) to model and simulate the nature of the concepts of limits.

Formative assessment tools were then given to learners to guide them throughout the remedial process. Unstructured interviews were also conducted with respondents to supplement the necessary insights and ideas gained by the students during the study. There was no analysis of the socio-economic variables of the students who participated in the study. The reasons behind the students’ attitudes towards mathematics were not examined and the analyses focused only on the comparison of the test scores results of the remedial classes.

With the help of the researcher-made models and simulations using DGS, the students were exposed to independent learning with the computer-assisted designed instructions. Dragging the objects (Figure 1) from the models in the drawing space of the DGS would provide the students with a real-time simulation of information updates regarding the different geometric relationships. In doing so, the students can efficiently test and confirm the different theoretical concepts of the topic. Worksheets were given to students to facilitate them in the entire remedial session. Afterward, the researcher-made posttest was administered to respondents to determine their understanding level of the lessons after the remediation process.

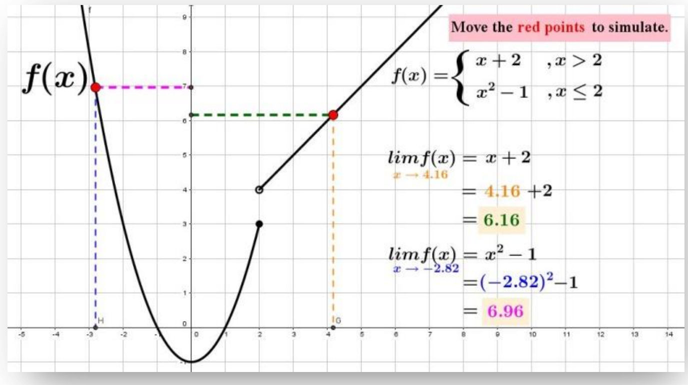

Figure 1. Modeling and simulation using GeoGebra

This research is descriptive and analytical in nature and used a Paired-samples *t-test* to determine if there is a significant improvement in students’ results before and after the remedial classes. This would determine if the use of modeling and simulation through GeoGebra would provide remedial measures to students who have problems understanding the concepts of limits in calculus. Unstructured interviews were also conducted to supplement the insights and ideas developed by the respondents during the remedial sessions.

3. RESULTS AND DISCUSSIONS

Given a graph of the piece-wise function (Figure 2), the student's skills in identifying the limit of a function on a specific x-value location were tested both in the pretest and posttest. Test items were formulated to test the students on identifying one-sided limits both from the left and right directions. Questions on finding the function value on a specific x-value location were also included as these are necessary to answer the items on determining the limit of a function and the type of discontinuity if there is any.

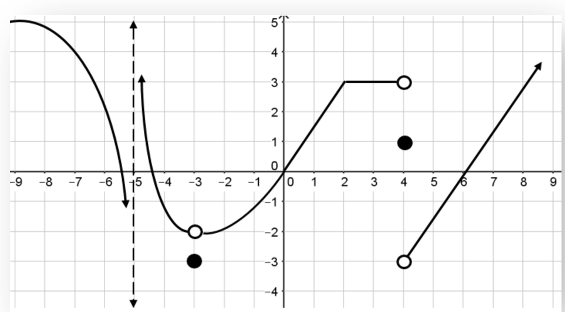

Figure 2. Graph of piece-wise function

Results from the pretest (Table 1) revealed that the students have an *Average Mastery* (53.63%) in identifying function values of functions when given a graph. At the x = 2 location, where the function is continuous, the students got the highest rating (77.42%) on identifying a function value. On the locations where there is a jump discontinuity (at x =

4) and a removable discontinuity (at x = -3), the students only got 68.55% mastery on average. However, everyone flunked on identifying a function value on a location where there is an infinity discontinuity. After the remediation with the help of modeling and simulations using the GeoGebra, their

Table 1. Students' achievement on limits before and after the remediation using GeoGebra

	Pretest			Posttest		
	f	%	QD	f	%	QD
Function Value						
f (2)	48	77.42	MT	62	100	M
f (4)	38	61.29	AM	58	93.55	CA
f (-3)	47	75.81	MT	50	80.65	MT
f (-5)	0	0	AN	61	98.39	M
Overall	33.25	53.63	AM	57.75	93.15	CA
One-sided Limits from left						
$lim_{x\to4^-} f(x)$	19	30.65	LM	59	95.16	CA
$lim_{x\to-3^-} f(x)$	27	43.55	AM	41	66.13	MT
$lim_{x\to-5^-} f(x)$	16	25.81	LM	52	83.87	MT
Overall	20.67	33.33	LM	50.67	81.72	MT
One-sided Limits from right						
$lim_{x\to4^+} f(x)$	24	38.71	AM	62	100	M
$lim_{x\to-5^+} f(x)$	16	25.81	LM	50	80.65	MT
Overall	20	32.26	AM	56	90.32	CA
Limit of a Function						
$lim_{x\to4} f(x)$	0	0	AN	44	70.97	MT
$lim_{x\to-3} f(x)$	33	53.23	AM	58	93.55	CA
$lim_{x\to-5} f(x)$	6	9.68	VL	59	95.16	CA
Overall	13	20.97	LM	53.67	86.56	CA
On proving Jump discontinuity						
$f(4)$	38	61.29	AM	58	93.55	CA
$lim_{x\to4^+} f(x)$	24	38.71	AM	62	100	M
$lim_{x\to4^-} f(x)$	19	30.65	LM	59	95.16	CA
$lim_{x\to4} f(x)$	0	0	AN	44	70.97	MT
…at what x-coordinate?	49	79.03	MT	56	90.32	CA
Overall	26	41.94	AM	55.8	90	CA
On proving Infinity discontinuity						
$lim_{x\to-5^-} f(x)$	16	25.81	LM	52	83.87	MT
$lim_{x\to-5^+} f(x)$	16	25.81	LM	50	80.65	MT
$lim_{x\to-5} f(x)$	6	9.68	VL	59	95.16	CA
…at what x-coordinate?	0	0	AN	61	98.39	M
Overall	9.5	15.32	LM	55.5	89.52	CA
On proving Removable discontinuity						
$lim_{x\to-3^-} f(x)$	27	43.55	AM	41	66.13	MT
$f(-3)$	47	75.81	MT	50	80.65	MT
$lim_{x\to-3} f(x)$	33	53.23	AM	58	93.55	CA
…at what x-coordinate?	37	59.68	AM	59	95.16	CA
Overall	36	58.07	AM	52	83.87	MT

Note: f = frequency of correct responses (out of 62), QD = Qualitative description; 0% - 4% = Absolutely No Mastery (AN), 5% - 14% = Very Low Mastery (VL), 15- 34= Low Mastery (LM), 35%-65%= Average Mastery (AM), 66%-85% = Moving towards Mastery (MT), 86%-95% = Closely Approximating mastery (CA), 96%-100% = Mastered (M)

achievement reached to *Closely Approximating Mastery* (93.15%). The student's understanding of function values was strengthened and notable increments on each item were observed in the post-test results especially on identifying function values on a location with infinity discontinuity.

The ability of the students to determine a limit of a function, involving the identification of the one-sided limits both from left to right was also improved from *Low Mastery* (20.97%) to *Closely Approximating Mastery* (86.56%). Students' confusion regarding function values at jump and infinity discontinuity locations was addressed with the help of the remediation using GeoGebra modeling and simulations. The real-time feedback of the software as students drag objects provided them with a concrete visual scenario of how limits behave in such locations [5].

On proving discontinuities of a certain function at a specific location, students need to determine the function value, one-sided limits, and how these values relate to each other to prove discontinuities. Before remediation, students have the lowest performance on identifying discontinuity at infinity (15.32%) while having an *Average Mastery* on both jump and removable discontinuities. After the series of activities with GeoGebra, students were able to obtain *Closely Approximating Mastery* (90%) and *Moving towards Mastery* (83.87%) levels on the jump and removable discontinuities, respectively.

Table 2. Paired-samples t-test on students' achievement on limits before and after remediation with GeoGebra

	M	SD	df	t	p-value	Cohen's δ
Pretest	6.02	2.68	61	-19.99	0.00*	2.54
Posttest	13.39	1.35				

* Significant at 0.05 level; δ = effect size (Cohen, 1988): 0.20 = small, 0.50 = medium, >0.80 = large.

A paired-samples t-test (Table 2) also revealed that the student's achievement on the concepts of limits in Calculus significantly improved after the remediation conducted with the use of GeoGebra. Additionally, the large effect size (δ=2.54) of the pre-experimental design can be accounted for by the intervention made to address the difficulty of the students in conceptualizing limits.

4. CONCLUSION AND RECOMMENDATIONS

The modeling and simulations using GeoGebra provided the students with an interactive environment where tasks of varying degrees of difficulty can be explored. The user-friendly and easy-to-grasp tools of the software can provide teachers and students with the necessary visual models that mimic the real-time scenario of how objects and their properties behave [6]. The mathematical representation ability of the students who were trained using Dynamic Geometry Software (DGS) like GeoGebra was better than those who were not [7]. Several studies also confirmed that GeoGebra-aided learning strategies were more effective than using the conventional or expository method of teaching. Utilizing mathematical interactive software provides a positive impact on enhancing students' learning and understanding of Mathematics. The remediation conducted to the students to address their confusion on understanding limits in Calculus provides a discovery activity that allows the students to manipulate the objects. The real-time feedback provided by the drawing interface of GeoGebra has provided the students the insights into how objects behave in geometrical space, bridging their understanding in locating the values shown by the computational algebraic processes. The results of the study may serve as a basis for the academic community to design workshops and learning action cell sessions on how to improve the teaching and learning of Mathematics across all areas.

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